

Vergnaud's Additive Conceptual Field: an Overview of Brazilian Academic Dissertations from 2000 to 2021

*O campo conceitual aditivo de vergnaud:
um panorama de dissertações acadêmicas brasileiras no período de 2000 a 2021*

*El campo conceptual aditivo de Vergnaud:
un panorama de las disertaciones académicas brasileñas de 2000 a 2021.*

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Abstract: This research aimed to carry out a mapping of the Brazilian academic dissertations whose main focus was the Additive Conceptual Field proposed by Gérard Vergnaud. The methodology adopted was the State of the Art, using Google Scholar as a source, in the period from 2000 to 2021. On this basis, the keyword used was 'Vergnaud' and 'Additive Field', in which the results were located in the surroundings of Teaching and Education area of basic schooling. Twenty-two academic dissertations were found in this search. Academic dissertations in Education were predominant (54.53%) in fifteen Pos Graduate programs. The predominant school segment of the surveys were the early years of Elementary School (86.36%). As for the research subject, dissertations involving students were the most frequent (68.18%), followed by teachers (13.63%), textbooks (9.09%) and curriculum material (9.09%).

Keywords: The State of art. Vergnaud. Conceptual field. Additive structures.

Resumo: Esta pesquisa objetivou realizar um mapeamento das dissertações acadêmicas brasileiras que tiveram como foco principal o Campo Conceitual Aditivo proposto por Gérard Vergnaud. A metodologia adotada foi o Estado da Arte, utilizando como fonte o Google Acadêmico, no período de 2000 a 2021. Nesta base a palavra-chave empregada foi 'Vergnaud' e 'Campo Aditivo', em que os resultados se situaram no entorno na área do Ensino e Educação da escolaridade básica. Nesta busca foram encontradas vinte e duas dissertações acadêmicas. As dissertações acadêmicas em Educação foram predominantes (54,53%), em quinze programas de Pós-Graduação. O segmento escolar predo-

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minante das pesquisas foram os anos iniciais do Ensino Fundamental (86,35%). Quanto ao sujeito de pesquisa, as dissertações envolvendo alunos foram as mais frequentes (68,18%), seguido de professores (13,63%), livro didático (9,09%) e material curricular (9,09%).

Palavras-chave: Estado da Arte. Vergnaud. Campo Conceitual. Estruturas aditivas.

Resumen: Esta investigación tuvo como objetivo llevar a cabo un mapeo de las disertaciones académicas brasileñas cuyo enfoque principal era el Campo Conceptual Aditivo propuesto por Gérard Vergnaud. La metodología adoptada fue el Estado del Arte, utilizando como fuente Google Scholar, en el período de 2000 a 2021. Sobre esta base, la palabra clave utilizada fue 'Vergnaud' y 'Additive Field', en la que los resultados se ubicaron en el entorno en el área de Magisterio y Educación de la escolaridad básica. En esta búsqueda se encontraron veintidós disertaciones académicas. Las disertaciones académicas en Educación predominaron (54,53%) en quince programas de Posgrado. El segmento escolar predominante de las encuestas fueron los primeros años de la Enseñanza Fundamental (86,35%). En cuanto al tema de investigación, las disertaciones de estudiantes fueron las más frecuentes (68,18%), seguidas de los profesores (13,63%), los libros de texto (9,09%) y el material curricular (9,09%).

Palabras clave: Estado del arte. Vergnaud. Campo conceptual. Estructuras aditivas.

Introduction

Gérard Vergnaud graduated in Psychology from the University of Paris (Sorbonne). The researcher made contributions to the Educational field by proposing a theoretical framework - the theory of Conceptual Fields - in which he discusses the additive and multiplicative structures in the Basic school segment.

Gérard Vergnaud actively acted in France for the creation of research groups in the field of mathematical education, which makes him seen, along with Guy Brousseau, as one of the pillars of the well-known and relevant movement of Didactics of Mathematics (MORO, s.d. p. 2).

In this text we are proposing to report the contributions of the Additive Field to Mathematical Education. Initially, considering the brazilian national curriculum parameters for the teaching of mathematics, according to Brazil (1997), we find indication to the Additive Field described in Gérard Vergnaud's Conceptual Field theory² as a bibliographic reference.

In the aforementioned text there is an explicit mention of the Additive Field, in several examples that explain the meanings from the natural numbers' addition and subtraction, having as support Vergnaud's ideas, since these operations "[...] make up the same family, in other words, there are narrow connections between additive and

2. The text mentioned and used in the brazilian National Curriculum Parameters, according to Brazil (1997), was VERGNAUD, G.; DURAND, C. Estructuras aditivas y complejidad psicogenética. Traduction from Reyes Villalonga. *Révue Française de Pédagogie*, 1976.

subtractive situations” (BRASIL, 1997, p. 69).

Given the importance of the theme for the Mathematics teaching area at basic education, this text aimed to carry out a mapping of the Brazilian academic dissertations whose main focus was the Additive Conceptual Field proposed by Gérard Vergnaud.

The theory of conceptual fields and the additive field

The Concept Field theory, from Vergnaud (1994), has as a central hypothesis the cognitive development based on conceptualization, which would be the cornerstone of cognition.

Teaching directed to formalization is necessary. However, it is necessary to take into account that scientific ideas evolve in the student during a long period of cognitive development, through a variety of situations and activities, and that any formal and axiomatized knowledge that the student presents may be no more than the visible part of an iceberg formed basically by implicit knowledge (VERGNAUD, 1990, p. 21).

Moreira (2002) and Rosa (2019) point out that the Vergnaud’s theory of the Conceptual Fields has a neopiagetian cognitivist nature. However, in a different way of Piaget, Vergnaud considered the own contents of knowledge and the conceptual analysis of its domain.

Vergnaud’s studies have redirected “[...] the Piagetian focus of general logical operations, the general structures of thought, to study the cognitive functioning of the subject-in-situation” (MOREIRA, 2002, p. 7).

Vergnaud himself (1996) states that the theory of conceptual fields:

- Has no intention of being a didactic theory, but providing a theoretical picture that allows us to understand the affiliations and ruptures between knowledge. Considers knowledge a competence (an appropriate action to treat the situation) or a conception (sequence of statements).

A maior parte dos nossos conhecimentos são competências e estas se formam, se desenvolvem, se diferenciam e, eventualmente, se deterioram ao longo de nossa experiência (p. 8).

Most of our knowledge are competences and these ones form, develop, and differentiate themselves and, eventually deteriorate, throughout our experience (p. 8).

To Vergnaud (1994), the school oversees explicit knowledge and underestimated (or devalues) the implicit knowledge of students. This author considers that knowledge-in-act (implicit) may evolve, over time, to scientific knowledge (explicit).

Vergnaud (1996) postulates that the meaning that a subject acquires is obtained through

classroom situations/tasks. The author considers that there are two types of situations: (i) the individual has the necessary competences for the immediate treatment of the situation, in an automated conduct that mobilizes a unique scheme (which the author called 'habit') (ii) the individual does not have all the necessary competences, which obliges him to reflect, to exploit and to hesitate, which makes him mobilize various schemes to solve the situation.

For Brun (1996), the scheme is the element that "[...] organizes and gives sense, simultaneously, to the actions, situations and symbolic representations that accompany them" (p. 23).

Vergnaud (1994) considers a scheme always rests on an implicit conceptualization. For the author, the schemes are composed of: (a) goals, rules of action and anticipations, information and control taking, which can generate a sequence of actions aimed at achieving a certain objective; (b) possibilities for inferences, which can be mobilized in each particular situation or set of situations; (c) operational invariants (concepts in action and theorems in action).

Vergnaud (1996b) considers that the schemes necessarily refer to situations, to such an extent that, according to him, one should speak of interaction scheme-situations rather than subject-object interaction of which Piaget spoke of. It follows that cognitive development consists, above all, and especially in the development of a vast repertoire of schemes (MOREIRA, 2002, p. 12).

Besides, Vergnaud (1994) highlights the existence of two fundamental schemes to be worked in Mathematics: (a) join, separate and the one-to-one correspondence, which corresponds to the additive structure; (b) distribute and put in one-to-many correspondence, which refers to the multiplicative structure.

To Vergnaud (1982, p. 40 *apud* MOREIRA, 2002) the conceptual field is "[...] an informal and heterogeneous set of problems, situations, concepts, relationships, structures, contents and thought operations, connected to each other and, probably, intertwined during the acquisition process" (p. 8).

Moreira (2002) highlights that the concept of conceptual field was developed by Vergnaud due to the fact that:

a concept do not form itself within one kind of situations; (2) a situation is not analyzed only with one concept; (3) The construction and appropriation of all the properties of a concept or all aspects of a situation is a very breathtaking process that extends over the years, sometimes a dozen years, with analogies and misunderstandings between situations, among conceptions, between procedures and signifiers (p. 8).

Vergnaud (1996) defines conceptual field as a set of situations that make up a variety of concepts, procedures and symbolic representations, that is, it corresponds

to triple of sets (S, I, R), where: (a) 'S' is a set of situations, which gives meaning to the object in question (the reference); (b) 'I' is a set of invariants, which deals with the properties and procedures necessary to define this object; it involves the relationship concepts and theorems-in-action (the meaning); (c) 'R' is a set of symbolic representations, which allow to relate the meaning of this object to its properties (the signifier).

The Additive Field, as reported by Vergnaud (1994), considers six categories, described as follows: (1) composition (situations where part and all are presented); (2) transformation (situations with three components - an initial state, a transformation and an end state); (3) comparison or comparison ratio (situations containing a relationship between two quantities, where one will be the referent and the referred to); (4) composition of two transformations (problems where two transformations occur and the third one is sought through a composition); (5) transformation of a relationship (situations where a relationship undergoes a transformation in order to emerge a new relationship); (6) composition of two relationships (where a new relationship is obtained from two initially given relationships).

Next, we presented the methodology, procedures and results of the empirical research that aimed to carry out a mapping of the Brazilian academic dissertations whose main focus was the Additive Field of the theory of Conceptual Fields designed by Gérard Vergnaud.

Empirical research: methodology and procedures

The methodology used was 'the State of the Art', which has as its prerogative to raise data, describe and systematically analyze the knowledge developed on a particular theme or area regarding the academic material already produced in a certain time interval, according to Soares (1989).

In Google Academic were inserted the words "Vergnaud" and "Additive Field". The repository indicated four hundred and thirty one (431) results in consultation on February 24, 2022, recorded under the site https://scholar.google.com.br/scholar?hl=pt-BR&as_sdt=0%2C5&as_vis=1&q=%E2%80%9Cvergnaud%E2%80%9D+and+%E2%80%9CCampo+aditivo%E2%80%9D+&btnG, excluding citations and patents, from 2000 to 2021.

After the floating reading of the summaries, we found twenty two (22) academic dissertations regarding the Additive Field that is related to Gérard Vergnaud's Conceptual Field theory. This material is referenced in Appendix A.

In Table 01 we indicate a synthesis of the twenty two (22) found results involving the Additive Field of Gérard Vergnaud.

Table 01: Synthesis of the 22 dissertations involving the Additive Field of Vergnaud.

	Dissertation	Year	Title
1	Alves	2016	There is an hour that we have learned to count by our head: a study on the construction of the number and the additive field in Early Childhood Education.
2	Araújo	2015	Additive Problems: A teaching proposal in the context of the game 'Steals Mount'.
3	Azevedo, K.	2017	Board game with RPG elements "Adventure of a Magic Book": contributions to mathematical education.
4	Azevedo, S.	2019	The challenge of arguing in Mathematics classes: an investigation with first year students of Elementary School.
5	Beck, M.	2019	Additive field in the set of integers: a study from the theory of Conceptual Fields.
6	Beck, V.	2015	Additive problems and algebraic thinking in the alphabetization cycle.
7	Brasil	2015	Analyzing the mobilization of pedagogical knowledge of the 3rd year teacher (early years) in the field of Additive Structures.
8	Costa	2011	Mathematical learning of everyday life: action strategies in marble game.
9	De Souza	2017	Alphabetization and Mathematical Literacy: perspectives and relationships between PNAIC and the textbook.
10	Dorneles	2013	Addition, subtraction and relational calculation: an intervention with students from PROEJA FIC/Elementary School.
11	Faxina	2017	Problem solving and the teaching of arithmetic concepts: perceptions of teachers from the early years of Elementary School.
12	Franzosi	2018	Grouping and disagreements in the Multibase Application: A proposal to teach the concept of number and operations of the additive conceptual field.
13	Furtuoso	2016	Analysis of the difficulties of oral respiratory students in problem solving of the additive concept.
14	Giombelli	2016	Implications of PNAIC formation on teachers' understandings on the elaborations of mathematical concepts by children of the alphabetization cycle.
15	Kehler	2012	As numbers and operations are approached in textbooks of the mathematical alphabetization phase.
16	Lima	2018	Algebraic thinking in the literacy cycle curriculum: comparative study of two proposals.
17	Oliveira	2010	Technology in Mathematical Education: the use of different resources for understanding the decimal numbering system (SND).
18	Pavan	2010	The mobilization of the basic ideas of the concept of function by children in the 4th grade of Elementary School in problem situations of Additive and/or Multiplicative Structures.
19	Reis	2017	Games, oral records and graphs: child development in the additive conceptual field.
20	Rocha	2019	Additive field problem solving strategies: an approach from the perspective of Conceptual field theory.
21	Silva, G.	2014	Conceptual Field theory, skills and competencies: a teaching experience in Mathematics.
22	Silva, L.	2015	Resignifying the construction of the addition and subtraction algorithms.

Font: The author

Next, we started to expose the results derived from academic dissertations.

Descriptions and results

For the presentation of the results we highlight some aspects of the main contri-

butions of the monographs analyzed.

Alves (2016) conducted an empirical research of interventionist and exploratory type following the principles of almost experimental research, with 5-year old students, whose objective was “[...] to present an analysis of the process of number construction and identifying its articulation with the main concepts of the Additive Field” (p. 7). The author’s proposal stood in three blocks, in which the first block was “[...] composed of jokes from the oral tradition and group games; the second block, with track game matches and the third block with the problem solving prototypes of the additive field ”(p. 7).

In this motto, Alves (2016) could work on the concept of counting through the possibility of kindergartens made their own strategies in facing of games and problems presented by the researcher. Alves (2016) found that the most used strategies were counting all and counting both.

In the second dissertation, Araujo (2015) applied an interventionist (almost experimental) research with pretest, intervention and final evaluation (post-test), involving 3rd grade students in the context of a game. The students used counting, overcount, complementation, pre-algorithms and algorithms as strategies. The author mentions that the game used contributed, a little, to improving children’s main difficulties: reciting numerical sequences, learning addition and subtraction techniques, as well as understanding additive problems.

In turn, the monograph of Azevedo, K. (2017) used design methodology, which sought to analyze the “[...] contributions from the board game with RPG elements ‘Adventure of a magic book’ to work with problems of additive structures” (p. 6). The research subjects were students of the 4th grade of Elementary School, with empiria through pretest, game experience and post-test. The classification was based on Magina *et al.* (2001), which groups Vergnaud’s typology of six additive problems into three: composition, positive and negative transformation and comparison.

In the fourth dissertation, Azevedo, S. (2019) used the additive field as a theoretical reference and had as its guiding question: “A didactic sequence that articulates the solving problems of Additive Structures and different moments of discussion in the classroom favors the development of argumentative skills by students of the 1st year of Elementary School?” (p. 7).

Azevedo, S. (2019) used Vergnaud’s theory as a reference for the organization of the didactic sequence. The research methodology was Didactic Engineering, according to Michelle Artigue. Going beyond the goal, the dissertation of Azevedo, S (2019)

made a praxeological analysis of textbooks of the 1st year of elementary school in the molds of Yves Chevallard.

The research of Beck, M. (2019) aimed to analyze textbooks and producing a sequence of classes on integers in the 7th grade of Elementary School. The research was declared qualitative, without further details.

The study presented by Beck, V. (2015) aimed to:

[...] analyze the main strategies and procedures of students of the 3rd year of the Alphabetization Cycle in solving problems that involve the competences and descriptors foreseen in the reference matrix for the evaluation of the Brazilian Mathematics Test [and] understand how algebraic thinking can be present in the resolution of additive problems (p. 6).

The methodology used in the research by Beck, V. (2015) was the School Action Investigation, in which the Theory of Conceptual Fields was the basis for evaluating the algebraic potential of some additive problems in two categories: the search for the unknown value and the results forecast.

The Monograph of Brazil (2015) aimed “[...] to analyze the mobilization of pedagogical knowledge of the 3rd year teacher [...] graduates of the Pedagogy Degree course at a public university in the city of Recife, in relation to the approach of the concept of additive structures” (p. 8). The chosen theoretical support was the categorization of Lee Shulman’s teaching knowledge and Gérard Vergnaud’s Conceptual Field theory. Brazil (2015) concluded that the teaching action regarding to the additive structures of the analyzed teachers reflected the model of the educational organization system adopted in the school.

The eighth monograph, by Costa (2011), aimed to “[...] analyze the influence of the categories of additive conceptual fields on the action strategies used by students in their resolution of mathematical problems and in different everyday situations” (p. 17). The methodological part consisted of applying a pre-test, with questions related to additive problems, followed by marbles.

The survey conducted by De Souza (2017) intended “[...] to analyze the convergences and divergences between the guidelines of PNAIC/2014 notebook 4 [...] and a collection of mathematical textbooks intended for the literacy cycle (1st, 2nd and 3rd years), focusing on the problems of additive and multiplicative structure” (p. 7). The theoretical frameworks used were Vergnaud’s Conceptual Field theory and the Chevallard’s Anthropological Didactic theory. The author concluded that there was a partial consonance between the two materials involved, and some types of problems

categorized by the Conceptual Field theory were not developed in PNAIC notebook 4 or the textbook collection analyzed.

Dorneles (2013) applied an intervention-research, with pre-test, intervention and post-test with students from the Freshwater Artisanal Fishing course and the Kitchen Assistant course. The objective of the research was “[...] to verify the role of teaching in learning the inverse relationship between addition and subtraction and whether relational calculus can help in understanding this inverse relationship” (p. 6). In this sense, the “[...] four intervention sessions were insufficient in relation to the needs presented by the students, and to obtain a more effective result, with adults, we suggest a greater number of interventions” (p. 7).

Faxina’s monograph (2017) aimed to “[...] investigate the perceptions that the teacher of the early years of elementary school has about working with solving problems in teaching arithmetic concepts” (p. 6). The data collected by Faxina (2017) indicated that some teachers “[...] has good ideas about working with problem solving and, especially, the importance of this work in the early years” (p. 6). Still, the author found that the difficulties of the teachers were “[...] related to the understanding of the decimal numbering system and the algorithms” (p. 6).

In the 12th dissertation, Franzosi (2018) conducted empirical research to elaborate and apply a sequence of activities based on games for the 2nd year of Elementary School whose objective was “[...] to analyze as groupings and disagreements of the virtual parts of the Multibase application can contribute to the teaching of the concept of numbers, decimal numbering system and additive field operations” (p. 8).

Furtuoso (2016) developed research in the area of Special Education that aimed to analyze “[...] the difficulties of operated and non-operated mouth breathing students in solving problems in the additive conceptual field” (p. 6). The author applied a sequence of Vergnaud’s additive problems to students in the 4th year of Elementary School with an experimental group and a control group.

Giombelli (2016), in the fourteenth monograph, aimed “[...] to verify if and in what sense the formation of PNAIC contributed to the teachers to understand how mathematical concepts are elaborated by children of the Elementary School alphabetization cycle” (p. 6). The theoretical foundation was based on Historical-Cultural Psychology and the Psychology of Mathematical Education (Gérard Vergnaud and Raymond Duval).

For data collection Giombeli (2016) used a participants profile survey (teachers) and problems involving additive structures (transformation, composition and

comparison), which had previously been resolved by a group of students. Giombelli (2016) reported that the survey instrument revealed that teachers who did not participate in the PNAIC presented some fragility in didactic knowledge related to the additive structures. On the other side, the group of teachers that participated in PNAIC revealed:

[...] greater awareness of the need for intervention in face of possible errors; greater concern in listening to the child's views and regulating interventions from them; a greater understanding that problems that involve understanding mathematical relationships and cannot be reduced only to linguistic issues; greater recognition of the need and the importance of using different forms representation (p. 6).

Kehler's (2012) dissertation aimed "[...] to investigate the treatment given to the 'natural numbers and decimal numbers system' and 'natural numbers' system in textbooks in the mathematical alphabetization phase, adopted by schools from State public network of Cuiabá city" (p. 8).

The analysis consisted of comparing two didactic collections, with the criterion of the structure of the works and the way of building the concept of literacy. Kehler's (2012) analysis revealed that the two collections considered aspects of "[...] historicization, contextualization and entanglement, as well as the use of games, games, text production and use of the resource of mathematics history" (p. 6).

Lima (2018) aimed "[...] to investigate what is the approach given to algebraic thinking in the prescribed curriculum of the alphabetization cycle, which refers to the first three years of Elementary School" (p. 9). The author used as research corpus the Brazilian Common National Curriculum Base (BNCC) and the Curriculum Guidelines of Mathematics for the early years (OCMAI), from São Paulo, and the methodology was Bardin's Content Analysis.

Lima (2018) concluded that BNCC presents a conceptual approach with Early Algebra, seeking the development of algebraic thinking from the first year of schooling. In OCMAI the development of algebraic thinking appears implicitly. In common, in the alphabetization cycle stand out the exploration of standards and regularities in contexts that can be generalized, without the use of algebraic symbolic language.

Oliveira (2010) conducted a research whose objective was "[...] contributing to the teaching of Mathematics and, recognizing the role that technologies can play in the teaching and learning process, we present a new resource that can help the teacher and the students in the SND study" (p. 6). The methodology consisted of applying an

intervention to three control groups: the first supports software developed by the researcher, the second using the abacus and the third through digital games.

Pavan (2010) conducted research that aimed “[...] to investigate whether children from the 4th grade of Elementary School recognize and mobilize elements of the conceptual field of function (such as variable, dependence, correspondence, regularity and generalization) in problem resolution situations of additive and/or multiplicative structures” (p. 6). Empiria used the Piagetian clinical method to raise which “[...] schemes used by children in solving problem situations and identify possible theorems-in-action investigated from the oral or gestural manifestations of children and the registration in problem situations” (p. 6).

The monograph of Reis (2017) aimed “[...] to analyze, from game situations, the oral and graphics records of children of the 3rd year of Elementary School in the additive field” (p. 7). The methodology was the case study and interventional research with children from the 3rd year of Elementary School.

The dissertation of Rocha (2019) investigated “[...] the strategies of students for solving additive problems from the 5th grade of Elementary School of public schools of the municipal education network” (p. 6). The research was designated as a case study with exploratory bias and the research instrument was a diagnostic test with ten problem situations addressing the concepts of composition, transformation and comparison. The results of the Rocha (2019) research indicated that:

[...] students used different strategies to try to solve the proposed problems, namely: algorithm of addition; subtraction algorithm; repetition of statement numbers; personal strategy, such as mental calculation and use of pictorial elements, in addition to the blank option, leaving the question without any outline of response. It was also found that even at the end of the early years of Elementary School (5th grade), students showed difficulties in dealing with additive field situations, except for prototypic situations (p. 6).

Rocha (2019) utilizou a classificação de Magina *et al.* (2001; 2008), que realizou uma releitura das seis categorias aditivas de Vergnaud (1996), apresentando subcategorias intituladas protótipos e extensões.

Silva, G. (2014) aimed to evaluate “[...] the contributions that a teaching experience, based on Vergnaud’s theory of Conceptual Fields, to students of the 3rd year of Elementary School in the development of competences and skills for resolution of problem situations” (p. 6). The methodology employed was the case study and the participating observation, in which the first phase was the application of a pretest, followed by a “[...] teaching experience to work on problem situations to better develop

those skills and skills desirable at this school level” (p. 6), which culminated with a post-test.

Silva, L. (2015) conducted a research that aimed “[...] to investigate the difficulties presented by a group of 2nd grade students [...], when performing the operations of addition and subtraction through algorithmic processes” (p. 12). The experimental part involved a poll, a diagnostic assessment of the difficulties reported in the survey, and had as finalization the “[...] construction of a didactic sequence, through an activity notebook, to assist teachers from the initial grades in the reconstruction of the algorithms of addition and subtraction through the resolution of problem situations involving additive ideas” (SILVA, L., 2015, p. 12).

Conclusions

After reading and synthetic description of dissertations, we began to synthesize and analyze the data. In Table 02 we indicate the distribution by area of postgraduate programs related to the surveyed monographs.

Table 02: Distribution by area of the 22 academic dissertations.

Areas	Monographs	Quantities
Education	Alves (2016); Beck, M. (2019); Beck, V. (2015); Costa (2011); Dornelles (2013); Faxina (2017); Furtuoso (2016); Giombelli (2016); Kehler (2012); Oliveira (2010); Reis (2017); Silva, G. (2014)	12
Education, Culture and Communication	Araujo (2015)	1
Education in (of) Science and Mathematics	Franzosi (2018); Rocha (2019)	2
Mathematics Education	Azevedo, S. (2019); De Souza (2017); Lima (2018)	3
Mathematics Education and Technology	Azevedo, K. (2017)	1
Education for Science and Mathematics	Pavan (2010)	1
Science teaching	Brasil (2015)	1
Mathematics teaching	Silva, L. (2015)	1
Total	---	22

Font: The author

The academic dissertations in education were predominant in fifteen of twenty-two postgraduate programs (which represented 54.53%). As follows, the area of mathematical education (13.63%) and education in (of) science and mathematics (9.09%). Finally, the others (education, culture and communication, education in (for) science and mathematics, mathematical and technological education, education for science and mathematics, science teaching, mathematics teaching), with a developed

academic text (the which represented 4.55% each).

Next, Table 03 summarizes the universities where the research was conducted.

Table 03: List of postgraduate programs.

Higher education institution	Monographs	Quantities
La Salle University	Silva, G. (2014)	1
Federal Institute of Espirito Santo	Franzosi (2018)	1
Pontifical Catholic University of Sao Paulo	Azevedo, S. (2019); De Souza (2017); Lima (2018)	3
Pontifical Catholic University of Minas Gerais	Silva L. (2015)	1
University of Brasília	Reis (2017)	1
State University of Rio de Janeiro	Alves (2016); Araujo (2015)	2
University of Vale do Itajai	Costa (2011)	1
State University of Maringá	Furtuoso (2016); Pavan (2010)	2
São Paulo State University 'Julio de Mesquita Filho'	Faxina (2017)	1
Federal University of Fronteira Sul	Giombelli (2016)	1
Federal University of Alagoas	Rocha (2019)	1
Federal University of Mato Grosso	Kehler (2012)	1
Federal University of Pernambuco	Azevedo, K. (2017); Oliveira (2010)	2
Federal University of Rio Grande do Sul.	Beck, M. (2019); Beck, V. (2015); Dorneles (2013)	3
Federal Rural University of Pernambuco	Brasil (2015)	1
Total	---	22

Fount: The author

There is a distribution of thematized dissertations in the Vergnaud's Additive Conceptual Field in fifteen postgraduate programs. Of these publications stand out the Pontifical Catholic University of São Paulo (13.63%), Federal University of Rio Grande do Sul (13.64%), Rio de Janeiro State University (9.09%), State University of Maringá (9.09%) and Federal University of Pernambuco (9.09%).

The initial criterion for surveying the material was the school segment to which dissertation research was intended, as described in Table 04.

Table 04: The school segment of research.

The school segment of research	Quantities
I. Kindergarten	1
2- Elementary School (initial years).	19
3- Elementary School (final years).	1
4- Youth and Adults Education	1
TOTAL	22

Fount: The author

The predominant school segment of the research stood in the early years of Elementary School, in 19 of 22 monographs (86.35%), followed by Kindergarten (4.55%), youth and adults education (4.55%) and Final years of Elementary School (4.55%).

Then, for each segment of education, the research subject, theoretical and methodological references were raised. Table 05 indicates research subject against the various teaching segments of basic education.

Table 05: The research subject.

The research subject	Kindergarten	Elementary School (initial years)	Elementary School (final years)	Youth and Adults Education	Quantities
Students	Alves (2016)	Araujo, C. (2015); Azevedo, K. (2017); Azevedo, S. (2019); Beck, V. (2015); Costa (2011); Furtuoso (2016); Oliveira (2010); Pavan (2010); Reis (2017); Rocha (2019); Silva. G. (2014); Silva. L. (2015)	--	Dorneles (2013)	15
Teachers	--	Brasil (2015); Faxina (2017); Giombelli (2016)	--	--	3
Textbook	--	Kehler (2020)	Beck, M. (2019)	--	2
Curriculum material	--	De Souza (2017); Lima (2018)	--	--	2
TOTAL	1	18	1	2	22

Fount: The author

As for the research subject we found fifteen dissertations directed to students (68.19%), followed by three works with Elementary School teachers (13.63%), two monographs involving textbooks (9.09%) and another two with relation to curriculum material (9.09%).

Table 06 describes the main texts of Gérard Vergnaud's Conceptual Field theory are used in the theoretical and/or methodological framework in the monographs aimed at the students of Kindergarten, Youth and Adults Education and the early years of Elementary School (research subject).

Table 06: Vergnaud texts used in research with students.

References	Monographs
Vergnaud (1979)	Dorneles (2013)
Vergnaud (1982)	Araujo, C. (2015)
Vergnaud (1983)	Furtuoso (2016)
Vergnaud (1986b)	Araujo, C. (2015); Beck, M. (2019); Oliveira (2010)
Vergnaud (1986a)	Furtuoso (2016)
Vergnaud (1990)	Dorneles (2013); Beck, V. (2015); Oliveira (2010); Pavan (2010)
Vergnaud (1991a)	Alves (2016)
Vergnaud (1991b)	Oliveira (2010); Pavan (2010)
Vergnaud (1993)	Araujo, C. (2015); Furtuoso (2016)
Vergnaud (1994)	Araujo, C. (2015)
Vergnaud (1996a)	Araujo, C. (2015); Azevedo, K. (2017); Azevedo, S. (2019); Beck, M. (2019); Dorneles (2003); Furtuoso (2016); Rocha (2019); Silva, G. (2014)
Vergnaud (1996b)	Silva, G. (2014)
Vergnaud (1996c)	Azevedo, S. (2019); Dorneles (2013); Furtuoso (2016)
Vergnaud (1997)	Beck, V. (2015)
Vergnaud (2008)	Reis (2017)
Vergnaud (2009a)	Reis (2017)
Vergnaud (2009b)	Alves (2016); Araujo, C. (2015); Azevedo, K. (2017); Beck, V. (2015); Costa (2011); Dorneles (2013); Franzosi (2018); Furtuoso (2016); Pavan (2010); Reis (2017); Rocha (2019); Silva, L. (2015)
Vergnaud (2009c)	Reis (2017); Furtuoso (2016);
Vergnaud (2009d)	Furtuoso (2016)
Vergnaud (2011)	Araujo, C. (2015); Azevedo, K. (2017)
Vergnaud (2014)	Azevedo, S. (2019)

Font: The author.

Ainda, o livro 'A criança, a matemática e a realidade: problemas do ensino da matemática na escola elementar', de 2009, foi o mais empregado (12 em 22 das monografias, ou seja, 54,54%), seguido do capítulo 'A Teoria dos Campos Conceituais' (8 das 22 pesquisas, representando 36,36%), inserido do livro de Jean Brun (1996).

In research involving students as a research subject it was observed a diversity of academic texts from Gérard Vergnaud, as a source of theoretical and/or methodological reference. Still, the book 'The Child, Mathematics and Reality: Problems of Mathematics Teaching at Elementary School', from 2009, was the most employed (12 in 22 of the monographs, i.e., 54.54%), followed by the chapter 'The theory of concept fields' (8 of the 22 research, representing 36.36%), inserted from the book of Jean Brun (1996).

Final considerations

The Additive Conceptual Field discussed in Vergnaud's Conceptual Field theory is present in the current prescribed curriculum, the Brazilian Common National Curriculum Base, which puts as 'object of knowledge' to the early three years of Elementary School the work with "[...] problems involving different meanings of addition and subtraction (join, add, separate, remove)" (BRAZIL, 2017, p. 234, p. 238, p. 242), which matches the presuppositions of Gérard Vergnaud.

As a highlight regarding research involving students (15), we realized that they were directed to activities or problem situations involving the six categories to the Additive Conceptual Field proposed by Gérard Vergnaud (1994). Of these, a monograph was directed to kindergarten, two at the 1st year, three to the 2nd year, seven to the 3rd year, two at the 4th year, one to the 5th year and one to the 7th year.

These results show the straight of Gérard Vergnaud's research, which began around 1979 and still remains in a state of update against the prerogatives of the area of mathematical education.

Finally, we emphasize that the results confirmed the pragmatic side on Gérard Vergnaud in the proposal developed in the theory of Conceptual Fields, which has strong relevance and direct application at classroom. In addition, we emphasize that this theoretical framework still remains relevant and alive in the bulge of current academic research.

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Appendix A – Academic Dissertations.

ALVES, Fabíola de Souza. There is an hour that we have learned to count by our head: a study on the construction of the number and the additive field in Early Childhood Education. 2016. 107f. Dissertation (Master in Education) – State University of Rio de Janeiro, Duque de Caxias, 2016.

ARAÚJO, Claudia Gomes. Additive Problems: A teaching proposal in the context of the game ‘Steals Mount’. 2015. 185f. Dissertação (Master in Education, Culture and Communication) - State University of Rio de Janeiro, Duque de Caxias, 2015.

AZEVEDO, Kelly de Lima. Board game with RPG elements “Adventure of a Magic Book”: contributions to mathematical education. 2017. 130f. Dissertation (Master Mathematical and Technological Education) - Federal University of Pernambuco, Ceará. 2017.

AZEVEDO, Simone Aparecida dos Anjos. The challenge of arguing in Mathematics classes: an investigation with first year students of Elementary School. 2019. 259f. Dissertation (Master in Mathematical Education) - Pontifical Catholic University of Sao Paulo, Sao Paulo. 2019.

BECK, Miguel Melendo. Additive field in the set of integers: a study from the theory of Conceptual Fields. 2019. 197f. Dissertation (Master in Education) - Federal University of Rio Grande do Sul, Rio Grande. 2019.

BECK, Vinicius Carvalho. Additive problems and algebraic thinking in the alphabetization cycle. 2015. 74f. Dissertation (Master in Education) - Federal University of Rio Grande do Sul, Rio Grande. 2015.

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LIMA, José Roberto de Campos. Algebraic thinking in the literacy cycle curriculum: comparative study of two proposals. 2018. 80f. Dissertation (Master in Mathematical Education) - Pontifical Catholic University of Sao Paulo, Sao Paulo. 2018.

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